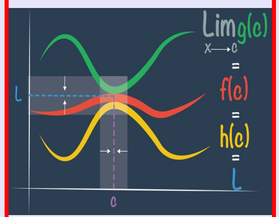
Calculus I

Lecture 21



Feb 19-8:47 AM

Class Quiz 9

Find $f'(x)$ for $f(x)=x^2$ using the definition of $f'(x)$, then evaluate $f'(-2)$.

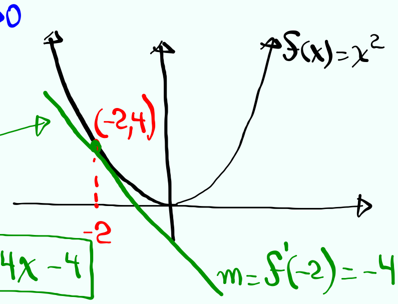
$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{(x+h)^2-x^2}{h}=\lim_{h\to 0}\frac{x^2+2xh+h^2-x^2}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{h}(2x+h)}{\cancel{h}}=\lim_{h\to 0}(2x+h)=2x+0=\boxed{2x}$$

$$f'(-2)=2(-2)=\boxed{-4}$$

$$y-4=-4(x-(-2))$$

$$y-4=-4x-8 \qquad \boxed{y=-4x-4}$$



Oct 3-7:03 AM

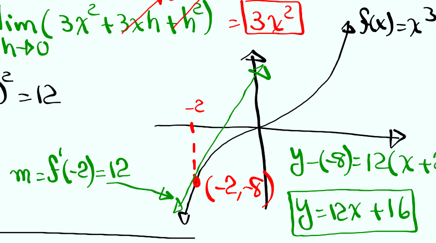
Find $f'(x)$ for $f(x)=x^3$ using def. of $f'(x)$ then evaluate at $x=-2$.

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{(x+h)^3-x^3}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{x^3}+3x^2h+3xh^2+h^3-\cancel{x^3}}{h}=\lim_{h\to 0}\frac{\cancel{h}(3x^2+3xh+h^2)}{\cancel{h}}$$

$$=\lim_{h\to 0}(3x^2+\overset{\to 0}{3xh}+\overset{\to 0}{h^2})=\boxed{3x^2}$$

$f'(-2)=3(-2)^2=12$



$y-(-8)=12(x+2)$

$\boxed{y=12x+16}$

$f(x)=x^2$	$f'(x)=2x$ ✓
$f(x)=x^3$	$f'(x)=3x^2$ ✓ $\Rightarrow f(x)=x^n$
$f(x)=x^4$	$f'(x)=4x^3$ ✓

$f'(x)=nx^{n-1}$

Oct 3-7:39 AM

Find $f'(x)$ for $f(x)=x^4$ using the def. of derivative.

$A^4-B^4=(A^2-B^2)(A^2+B^2)$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{(x+h)^4-x^4}{h}$$

$$=\lim_{h\to 0}\frac{\overbrace{[(x+h)^2-x^2]}^{A^2-B^2}[(x+h)^2+x^2]}{h}$$

$$=\lim_{h\to 0}\frac{[\cancel{x}+\cancel{h}-\cancel{x}][x+h+x][(x+h)^2+x^2]}{\cancel{h}}$$

$$=\lim_{h\to 0}\left[(2x+\overset{\to 0}{h})[(x+\overset{\to 0}{h})^2+x^2]\right]=2x(x^2+x^2)$$

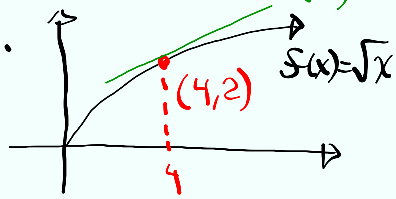
$$=2x\cdot 2x^2$$

$$=\boxed{4x^3}$$

Oct 3-7:50 AM

Find slope of the tan. line to the graph → $m = f'(4)$

of $f(x) = \sqrt{x}$ at $x = 4$.



$\boxed{f(x) = \sqrt{x}}$

$f(x) = x^{1/2}$ Using the Power Rule

$f(x) = x^n$, $f'(x) = n x^{n-1}$

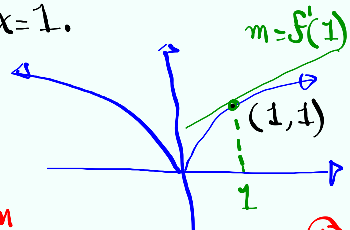
$f'(x) = \frac{1}{2} x^{\frac{1}{2}-1} = \frac{1}{2} x^{-1/2} = \frac{1}{2x^{1/2}} = \boxed{\frac{1}{2\sqrt{x}}} = \frac{\sqrt{x}}{2x}$

$f'(4) = \frac{1}{2\sqrt{4}} = \boxed{\frac{1}{4}}$

Oct 3-7:59 AM

Find equation of the tan. line to the graph

of $f(x) = \sqrt[3]{x^2}$ at $x = 1$.



$f(x) \geq 0$

$f(-x) = \sqrt[3]{(-x)^2} = \sqrt[3]{x^2} = f(x)$

$f(x)$ is an even function

symmetric w/t y-axis

$\frac{2}{3} - 1 = \frac{2}{3} - \frac{3}{3} = -\frac{1}{3}$

$f(x) = \sqrt[3]{x^2}$ $\quad f(x) = x^{2/3}$ Using Power rule

$f(x) = x^n$

$f(1) = \sqrt[3]{1^2} = 1$ $\quad f'(x) = \frac{2}{3} x^{-1/3}$

$f'(x) = n x^{n-1}$

$= \frac{2}{3x^{1/3}} = \frac{2}{3\sqrt[3]{x}}$ $\quad f'(1) = \frac{2}{3\sqrt[3]{1}}$

$\boxed{f'(1) = \frac{2}{3}}$

$y - y_1 = m(x - x_1)$

$y - 1 = \frac{2}{3}(x - 1) \rightarrow \boxed{y = \frac{2}{3}x + \frac{1}{3}}$

Oct 3-8:05 AM

Prove that if $f(x)=x^n$, then $f'(x)=nx^{n-1}$.

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{(x+h)^n-x^n}{h}$$

Look up Binomial exp. $(A+B)^n$

$$=\lim_{h\to 0}\frac{\cancel{x^n}+nx^{n-1}h+\frac{n(n-1)}{2}x^{n-2}h^2+\cdots+h^n-\cancel{x^n}}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{h}\left(nx^{n-1}+\underbrace{\cancel{\frac{n(n-1)}{2}x^{n-2}h+\cdots+h^{n-1}}}_{\to 0}\right)}{\cancel{h}}$$

$$=\boxed{nx^{n-1}}$$

Oct 3-8:16 AM

Find $f'(x)$ for any quadratic function

$f(x)=ax^2+bx+c$, $a\neq 0$

$$f'(x)=\lim_{h\to 0}\frac{\overbrace{a(x+h)^2+b(x+h)+c}^{f(x+h)}\ \overbrace{-ax^2-bx-c}^{-f(x)}}{h}$$

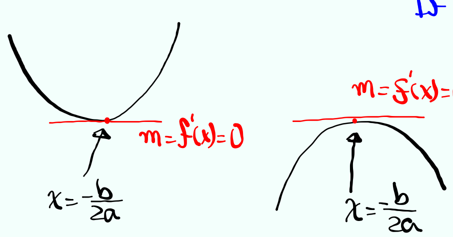
$$=\lim_{h\to 0}\frac{\cancel{ax^2}+2axh+ah^2+\cancel{bx}+bh+\cancel{c}-\cancel{ax^2}-\cancel{bx}-\cancel{c}}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{h}(2ax+\underbrace{\cancel{ah}}_{\to 0}+b)}{\cancel{h}}=2ax+b$$

If $f'(x)=0$

$2ax+b=0$

$x=\frac{-b}{2a}$



Vertex

$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

Oct 3-8:21 AM

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$f(x) = \sin x \to f'(x) = \cos x$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$f(x) = \cos x \to f'(x) = -\sin x$

$f(x) = x^n \to f'(x) = n x^{n-1}$

Find $\displaystyle \lim_{x \to 1} \frac{x^{13} - 1}{x - 1} = f'(1)$

$= \boxed{13}$

$f(x) = x^{13}$ $\quad a = 1$

$f(1) = 1$

$f'(x) = 13x^{12}$

$f'(1) = 13$

Evaluate $\displaystyle \lim_{x \to 4} \frac{\sqrt{x} - 2}{x - 4}$

$f(x) = \sqrt{x}$ $\quad a = 4$

$f(4) = 2$

$f'(x) = \dfrac{1}{2\sqrt{x}}$

verify by rationalizing the num.

$= f'(4) = \dfrac{1}{2\sqrt{4}} = \boxed{\dfrac{1}{4}}$

Oct 3-8:27 AM